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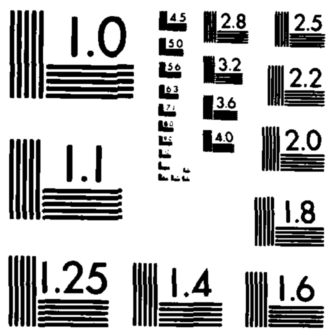

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RELATIONSHIPS BETWEEN PRIME-RICH EULER TYPE EQUATIONS AND A TRIANGULAR ARRAY OF THE ODD INTEGERS

AD-A157 928

BY R. S. SERY

RESEARCH AND TECHNOLOGY DEPARTMENT

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A certain triangular array of the odd integers, which is built up along successive diagonals of a rectangular grid, is correlated with equations of the form x^2-x+c, c=2n-1, n+1,2,3... where n indicates the number of the column (and also the row number for equations x^2+x-r). "Primitive" cell arrays are derived from the above array. Each such array consists of a prime value p(of c) which is substituted for every integer in the triangular array which is ≡0(mod p) with all other locations corresponding to integers not ≡(mod p) left blank.

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Various relationships between the triangular array, the equations x^2-x+c, $c=1,3,5,\ldots$ and the derived primitive cell arrays are brought out. These show how

1. Euler type prime-rich equations can be found.

2. Why and how non-random structure exists in the way prime numbers occur in the sequence of the positive integers 1,2,3, ...

FOREWORD

There is an increasing interest within the Navy and DoD in the study of prime numbers for applications in disciplines such as artificial intelligence and encription. This report describes a contribution to prime number theory. It involves the formation of a special diagonal array whose columns and rows can be generated by either of two sets of quadratic equations. The analysis of this array gives a clear picture of how and why there is a definite structure in the way prime numbers occur in the cardinal number system.

The analysis on which this report is based was done on the employee's own time and its publication was charged to overhead funds because of the relevance of the subject to the Navy.

Approved by:

J. R. Dixon

JACK R. DIXON, Head
Materials Division

CONTENTS

TABLES

INTRODUCTION

A 1964 Scientific American article[1] described Stanislaw Ulam's discovery in 1963 of the non-random distribution of prime numbers in a special array. The array consisted of a square spiral made up of the integers which started at the center of a rectangular grid. The primes tended to be aligned along straight lines, particularly diagonals. The article mentioned that various other types of arrays (not specifically described) also showed similar behavior.

This paper describes one such array which, to the author's knowledge, has not been reported before. It is built up of successive diagonals, each of which starts at the leftmost column and consists of the odd integers only (see Table 1). Each column and row can be represented by an equation of the form $I=x^2-x+c$ and $I=x^2+x-r$ respectively. The array can be built up in several ways such as directly by filling in successive diagonals (e.g., for the first 1, for the second 3,5, the third 7,9,11, etc.) or by substitution in each equation or by addition in each equation as follows: Calculate the first two integers in a column from an equation, then the third integer $I_3=I_2-I_1+I_2+2$ or $I_3=I_2+\delta+2$ where $\delta=I_2-I_1$. In general, $I_{n+1}=I_n+\delta+2$, where $\delta=I_n-I_{n-1}$ and n+1, n and n-1 indicate positions in the array. The first method can be faster but either of the other two has the advantage that it permits one to put the triangular array into rectangular form.

An initial array was built up of 75 columns by 40 rows. The limit of 40 rows was chosen because the best known prime-rich equation, x^2-x+41 (due to Euler) has 40 primes in succession for x=1,2,3,......,40. This is not evident in the original array of 75x40. Only 22 columns (but 40 rows) are shown in Table 1. A second partial table with 40 rows but only 11 columns does show these first 40 primes (Table 2). It is obvious that if one wishes to find other prime-rich equations and compare them to x^2-x+41, one must substitute those values of x that include 1,2,3,..... up to x_1 (the first value of x for each equation) in Table 1. Except for the first column (and first row) values of $x<x_1$ are increasingly excluded as c and r increase in value in Table 1.

One of the main advantages of using Table 1 was that in studying it, a correlation was found between prime-rich equations and certain characteristics they possess. It is strong enough to insure that most prime-rich equations of

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[1] Gardner, Martin, "Mathematical Recreations," Scientific American, Vol. 210, No. 3, Mar 1964, pp. 120-127.

TABLE 1. DIAGONALLY BUILT ARRAY (MODIFIED TO RECTANGULAR FORM) OF ODD INTEGERS ONLY (PRIMES ARE UNDERLINED)

$$I_c = x^2 - x + c$$

	c=1	3	5	7	9	11	13	15	17	19	21
	x_1=1	2	3	4	5	6	7	8	9	10	11
r=1	1	5	11	19	29	41	55	71	89	109	131
r=3	3	9	17	27	39	53	69	87	107	129	153
5	7	15	25	37	51	67	85	105	127	151	177
7	13	23	35	49	65	83	103	125	149	175	203
9	21	33	47	63	81	101	123	147	173	201	231
11	31	45	61	79	99	121	145	171	199	229	261
13	43	59	77	97	119	143	169	197	227	259	293
15	57	75	95	117	141	167	195	225	257	291	327
17	73	93	115	139	165	193	223	255	289	325	363
19	91	113	137	163	191	221	253	287	323	361	401
21	111	135	161	189	219	251	285	321	359	399	441
23	133	159	187	217	249	283	319	357	397	439	483
25	157	185	215	247	281	317	355	395	437	481	527
27	183	213	245	279	315	353	393	435	479	525	573
29	211	243	277	313	351	391	433	477	523	571	621
31	241	275	311	349	389	431	475	521	569	619	671
33	273	309	347	387	429	473	519	567	617	669	723
35	307	345	385	427	471	517	565	615	667	721	777
37	343	383	425	469	515	563	613	665	719	775	833
39	381	423	467	513	561	611	663	717	773	831	891
41	421	465	511	559	609	661	715	771	829	889	951
43	463	509	557	607	659	713	769	827	887	949	1013
45	507	555	605	657	711	767	825	885	947	1011	1077
47	553	603	655	709	765	823	883	945	1009	1075	1143
49	601	653	707	763	821	881	943	1007	1073	1141	1211
51	651	705	761	819	879	941	1005	1071	1139	1209	1281
53	703	759	817	877	939	1003	1069	1137	1207	1279	1353
55	757	815	875	937	1001	1067	1135	1205	1277	1351	1427
57	813	873	935	999	1065	1133	1203	1275	1349	1425	1503
59	871	933	997	1063	1131	1201	1273	1347	1423	1501	1581
61	931	995	1061	1129	1199	1271	1345	1421	1499	1579	1661
63	993	1059	1127	1197	1269	1343	1419	1497	1577	1659	1743
65	1057	1125	1195	1267	1341	1417	1495	1575	1657	1741	1827
67	1123	1193	1265	1339	1415	1493	1573	1655	1739	1825	1913
69	1191	1263	1337	1413	1491	1571	1653	1737	1823	1911	2001
71	1261	1335	1411	1489	1569	1651	1735	1821	1909	1999	2091
73	1333	1409	1487	1567	1649	1733	1819	1907	1997	2089	2183
75	1407	1485	1565	1647	1731	1817	1905	1995	2087	2181	2277
77	1483	1563	1645	1729	1815	1903	1993	2085	2179	2275	2373
79	1561	1643	1727	1813	1901	1991	2083	2177	2273	2371	2471

TABLE 1. (Cont.)

c=23	25	27	29	31	33	35	37	39	41	43
x_1=12	13	14	15	16	17	18	19	20	21	22
155	181	209	239	271	305	341	379	419	461	505
179	207	237	269	303	339	377	417	459	503	549
205	235	267	301	337	375	415	457	501	547	595
233	265	299	335	373	413	455	499	545	593	643
263	297	333	371	411	453	497	543	591	641	693
295	331	369	409	451	495	541	589	639	691	745
329	367	407	449	493	539	587	637	689	743	799
365	405	447	491	537	585	635	687	741	797	855
403	445	489	535	583	633	685	739	795	853	913
443	487	533	581	631	683	737	793	851	911	973
485	531	579	629	681	735	791	849	909	971	1035
529	577	627	679	733	789	847	907	969	1033	1099
575	625	677	731	787	845	905	967	1031	1097	1165
623	675	729	785	843	903	965	1029	1095	1163	1233
673	727	783	841	901	963	1027	1093	1161	1231	1303
725	781	839	899	961	1025	1091	1159	1229	1301	1375
779	837	897	959	1023	1089	1157	1227	1299	1373	1449
835	895	957	1021	1087	1155	1225	1297	1371	1447	1525
893	955	1019	1085	1153	1223	1295	1369	1445	1523	1603
953	1017	1083	1151	1221	1293	1367	1443	1521	1601	1683
1015	1081	1149	1219	1291	1365	1441	1519	1599	1681	1765
1079	1147	1217	1289	1363	1439	1517	1597	1679	1763	1849
1145	1215	1287	1361	1437	1515	1595	1677	1761	1847	1935
1213	1285	1359	1435	1513	1593	1675	1759	1845	1933	2023
1283	1357	1433	1511	1591	1673	1757	1843	1931	2021	2113
1355	1431	1509	1589	1671	1755	1841	1929	2019	2111	2205
1429	1507	1587	1669	1753	1839	1927	2017	2109	2203	2299
1505	1585	1667	1751	1837	1925	2015	2107	2201	2297	2395
1583	1665	1749	1835	1923	2013	2105	2199	2295	2393	2493
1663	1747	1833	1921	2011	2103	2197	2293	2391	2491	2593
1745	1831	1919	2009	2101	2195	2291	2389	2489	2591	2695
1829	1917	2007	2099	2193	2289	2387	2487	2589	2693	2799
1915	2005	2097	2191	2287	2385	2485	2587	2691	2797	2905
2003	2095	2189	2285	2383	2483	2585	2689	2795	2903	3013
2093	2187	2283	2381	2481	2583	2687	2793	2901	3011	3123
2185	2281	2379	2479	2581	2685	2791	2899	3009	3121	3235
2279	2377	2477	2579	2683	2789	2897	3007	3119	3233	3349
2375	2475	2577	2681	2787	2895	3005	3117	3231	3347	3465
2473	2575	2679	2785	2893	3003	3115	3229	3345	3463	3583
2573	2677	2783	2891	3001	3113	3227	3343	3461	3581	3703

TABLE 2. ARRAY OF TABLE 1 MODIFIED SO THAT ALL EQUATIONS HAVE x_1=1, i.e., x=1,2,3,....,40 TO ENABLE MORE ACCURATE COMPARISONS OF DENSITIES OF PRIMES TO BE MADE

c=...11	13	15	17	....	31	33	35	37	39	41
x_1*										
11	13	15	17		31	33	35	37	39	41
13	15	17	19		33	35	37	39	41	43
17	19	21	23		37	39	41	43	45	47
23	25	27	29		43	45	47	49	51	53
31	33	35	37		51	53	55	57	59	61
41	43	45	47		61	63	65	67	69	71
53	55	57	59		73	75	77	79	81	83
67	69	71	73		87	89	91	93	95	97
83	85	87	89		103	105	107	109	111	113
101	103	105	107		121	123	125	127	129	131
121	123	125	127		141	143	145	147	149	151
143	145	147	149		163	165	167	169	171	173
167	169	171	173		187	189	191	193	195	197
193	195	197	199		213	215	217	219	221	223
221	223	225	227		241	243	245	247	249	251
251	253	255	257		271	273	275	277	279	281
283	285	287	289		303	305	307	309	311	313
317	319	321	323		337	339	341	343	345	347
353	355	357	359		373	375	377	379	381	383
391	393	395	397		411	413	415	417	419	421
431	433	435	437		451	453	455	457	459	461
473	475	477	479		493	495	497	499	501	503
517	519	521	523		537	539	541	543	545	547
563	565	567	569		583	585	587	589	591	593
611	613	615	617		631	633	635	637	639	641
661	663	665	667		681	683	685	687	689	691
713	715	717	719		733	735	737	739	741	743
767	769	771	773		787	789	791	793	795	797
823	825	827	829		843	845	847	849	851	853
881	883	885	887		901	903	905	907	909	911
941	943	945	947		961	963	965	967	969	971
1003	1005	1007	1009		1023	1025	1027	1029	1031	1033
1067	1069	1071	1073		1087	1089	1091	1093	1095	1097
1133	1135	1137	1139		1153	1155	1157	1159	1161	1163
1201	1203	1205	1207		1221	1223	1225	1227	1229	1231
1271	1273	1275	1277		1291	1293	1295	1297	1299	1301
1343	1345	1347	1349		1363	1365	1367	1369	1371	1373
1417	1419	1421	1423		1437	1439	1441	1443	1445	1447
1493	1495	1497	1499		1513	1515	1517	1519	1521	1523
1571	1573	1575	1577		1591	1593	1595	1597	1599	1601

* x_1 = 1 for all columns

the form x^2-x+c can be found by its use (see Appendix A, Para. No. 4). The correlation is that a column tends to be rich in primes if c itself is prime and x_1 is $\equiv 0 \pmod 3$. Richness was defined as having a density of >50 percent where the density is the ratio of primes in a column divided by the number of integers in that column. In the array of Table 2, the denominator was fixed at 40 which gives for c=41 (i.e., x^2-x+41) a density of 100 percent. Other prime-rich equations found in the original 75x40 array were those of c=11,17,59,67. In addition, by using the criteria c=prime, $x_1 \equiv 0 \pmod 3$ and not extending the columns of the original array, the following prime-rich equations were found: c=101,107,137,227,251,257,311,347,353,359,419,431, etc. For brevity, the density for most of these was limited to ~60 percent (seven others did have a density $\geq$50 percent). There were equations which did not meet the criteria whose densities were $\geq$50 percent. They are: c=67,95, and 367 (see Table 3). Of these only c=95 is not prime (although $x_1=48$ is $\equiv 0 \pmod 3$). In contrast, the other two values of c are prime but the corresponding values of x_1 are not $\equiv 0 \pmod 3$. It is very likely that a number of similar examples will be found. It would be more feasible to look for them, if desired, by means of a suitable computer program rather than by the method that was used (hand calculator).

The total number of prime-rich equations found by this method was less than the total number of equations which met the criteria c=prime, $x_1 \equiv 0 \pmod 3$. This is a weakness of the correlation. Of the 75 columns originally studied 34 had c=prime. Of these, 18 fulfilled the criteria. The ratio of prime-rich equations to those meeting the criterion is only about 0.44, i.e., 8/18. Nevertheless, the use of the criterion does seem to offer a viable method of looking for prime-rich equations.

PRIMITIVE CELL ARRAYS DERIVED FROM SPECIAL ARRAY

There is another method derived from a study of the new array which might also prove useful for searching for them. This will be described below. To start with, Tables 4, 5, and 6 illustrate another interesting characteristic which can be inferred from the array of Table 1. If we look for each integer in the array for which $I \equiv 0 \pmod 3$ and replace it with the value 3 and leave all other integer locations blank, we can construct Table 4. Similarly, we get Table 5 for those integers $\equiv 0 \pmod 7$ and Table 6 for the integers $\equiv 0 \pmod{11}$. A table for the divisor 5 is not included but can be inferred from Table 7. The conjecture can be made, and is, that similar tables for all primes can be constructed in principle. It is apparent from Tables 4, 5, and 6 that there are repeating patterns of 3's, 7's, and 11's in the respective tables. One can borrow, with modifications, the concept of a primitive cell from Crystallography. There the smallest unit cell (meeting certain criteria which are not pertinent here) is called a primitive cell and the pattern of the whole crystal can be represented by it. Similarly one can build up each array of Tables 4, 5, and 6 by appropriate translations of the "primitive" cell. The concept of a basic pattern of a roll of wallpaper repeated indefinitely in two dimensions is also a useful analogy. Here the primitive cell is defined as that "area" of the array which is common to the first n rows and the first n columns

TABLE 3. THOSE EQUATIONS, $x^2 - x + c$, MEETING THE CRITERIA C IS PRIME AND $x_1 \equiv 0 \pmod 3$ WHOSE DENSITIES OF PRIMES EXCEED 50 PERCENT (WITH THREE EXCEPTIONS INCLUDED)

c	Is prime	x_1	Is $\equiv 0 \pmod 3$	Criteria satisfied	R,%*	Integers in column $\equiv 0 \pmod 5$
11	Yes	6	Yes	Yes	62.5	None
17	"	9	"	"	77.5	None
41	"	21	"	"	100.0	None
59	"	30	"	"	60.0	20%
67	"	34	No	No	62.5	None
95	No	48	Yes	No	50.0	37.5%
101	Yes	51	"	Yes	75.0	None
107	"	54	"	"	72.5	None
137	"	69	"	"	65.0	None
227	"	114	"	"	72.5	None
251	"	126	"	"	65.0	None
257	"	129	"	"	57.5	None
311	"	156	"	"	65.0	None
347	"	174	"	"	65.0	None
353	"	177	"	"	57.5	40%
359	"	180	"	"	57.5	20%
367	"	184	No	No	62.5	None
389	"	195	Yes	Yes	65.0	20%
419	"	210	"	"	55.0	20%
431	"	216	"	"	62.5	None
557	"	279	"	"	65.0	None
587	"	294	"	"	72.5	None

* $R = \frac{\text{No. of primes}}{\text{No. of integers}}$ in a column.

These ratios are based on letting $x_1=1$ (i.e., $x = 1,2,3,....,40$) as in Table 2.

TABLE 4. PRIMITIVE CELL OF 3 (IN BOX) AND ITS "TRANSLATIONS" IN THE c AND r DIRECTIONS AS CONSTRUCTED FROM TABLE 1 BY PUTTING 3's AT LOCATIONS WHOSE INTEGERS ARE $\equiv 0 \pmod 3$ AND LEAVING BLANK ALL OTHER LOCATIONS

x^2-x+c

c =	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41
r = 1																					
r = 3	3	3		3	3		3	3		3	3		3	3		3	3		3	3	
r = 5		3			3			3			3			3			3			3	
r = 7																					
r = 9	3	3		3	3		3	3		3	3		3	3		3	3		3	3	
r = 11		3			3			3			3			3			3			3	
r = 13																					
r = 15	3	3		3	3		3	3		3	3		3	3		3	3		3	3	
r = 17		3			3			3			3			3			3			3	
r = 19																					

TABLE 5. PRIMITIVE CELL OF 7 (in box) AND ITS TRANSLATIONS WITH LOCATIONS OF INTEGERS IN TABLE 1 WHICH ARE ≡0(mod 7) MARKED BY 7's WITH ALL OTHER LOCATIONS LEFT BLANK

C =	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43
x_1 =	1	2	3	4	.	.	.	.	.	.	.								.	.	.	. 22
= 1																						
2																						
3	7							7							7							7
4			7	7						7	7						7	7				
5	7			7				7			7				7			7				7
6																						
7			7		7					7		7					7		7			
8																						
9																						
10	7							7							7							7
11			7	7						7	7						7	7				
12	7			7				7			7				7			7				7
13																						
14			7		7					7		7					7		7			
15																						
16																						
17	7							7							7							7
18			7	7						7	7						7	7				
19	7			7				7			7				7			7				7
20																						
21			7		7					7		7					7		7			
22																						
23																						
24	7							7							7							7
25			7	7						7	7						7	7				
26	7			7				7			7				7			7				7

TABLE 6. PRIMITIVE CELL OF 11 (in box) AND ITS TRANSLATIONS WITH LOCATIONS OF INTEGERS IN TABLE 1 WHICH ARE ≡0(mod 11) MARKED BY 11's WITH ALL OTHER LOCATIONS LEFT BLANK

C =	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43
x_1 =	1	2	3	. . . .																		
= 1			11				11							11				11				
2																						
3																						
4																						
5		11									11		11									11
6					11	11										11	11					
7			11			11								11			11					
8																						
9					11						11					11						11
10							11											11				
11																						
12			11				11							11				11				
13																						
14																						
15																						
16		11									11		11									11
17					11	11										11	11					
18			11			11								11			11					
19																						
20					11						11					11						11
21							11											11				
22																						
23			11				11							11				11				
24																						
25																						
26																						

The curve shown is a parabola of the form $x^2+x+y^2+y-2xy=0$ with apex x=y=o at the center of a translation of the primitive cell.

TABLE 7. ANALYSIS WHICH SHOWS HOW THE PRIMITIVE CELL OF 5 IS REPEATED THROUGHOUT THE UNLIMITED ARRAY WHICH RESULTS FROM THE WAY IN WHICH INTEGERS IN TABLE 1 ARE FORMED FROM THE EQUATIONS x^2-x+c, c=1,3,5,....,2n-1,....

	+c=1*	=3	=5	=7	=9	=11	=13	=15	=17	=19
x(x-1)*	x_1=1	=2	=3	=4	=5	=6	=7	=8	=9	=10
1(0)=0	1	$\underline{5}$	1	9	9	1	5	1	9	9
2(1)=2	3	9	7	7	9	3	9	7	7	9
3(2)=6	7	$\underline{5}$	$\underline{5}$	7	1	7	5	5	7	1
4(3)=12	3	3	$\underline{5}$	9	$\underline{5}$	3	3	5	9	5
5(4)=20	1	3	7	3	1	1	3	7	3	1
6(5)=30	1	5	1	9	9	1	5	1	9	9
7(6)=42	3	9	7	7	9	3	9	7	7	9
8(7)=56	7	5	5	7	1	7	5	5	7	1
9(8)=72	3	3	5	9	5	3	3	5	9	5
10(9)=90	1	3	7	3	1	1	3	7	3	1
11(10)=110	1	5	1	9	9	1	5	1	9	9
12(11)=132	3	9	7	7	9	3	9	7	7	9
13(12)=156	7	5	5	7	1	7	5	5	7	1
14(13)=182	3	3	5	9	5	3	3	5	9	5
15(14)-210	1	3	7	3	1	1	3	7	3	1

*Note: Only digits parts of all integers are retained. The primitive cell for 5 is outlined in the box. 1 is added to 0,2,6,2,0 for the 1st column; 3 is added to 2,6,2,0,0,... for the 2nd; 5 is added to 6,2,0,0,2,... for the 3rd, etc.

for each c (and r) and taking c=r. For c=3, it is the area containing the nine integers 1,5,11; 3,9,17; and 7,15,25. For c=5, it is that area common to the integers in the area "bounded" by c=1,3,5,7,9 and r=1,3,5,7,9. It is assumed that each primitive cell can be "translated" endlessly in the c and r directions.

CHARACTERISTICS OF PRIMITIVE CELL ARRAYS

A little study of the primitive cells for 3,5,7,11, and 17 resulted in the observation of several properties of the primitive cell type arrays. These are summarized below. Only prime values of c need be considered as composite values of c are multiples of the prime values; but the properties apply to all c's and r's whether prime or composite.

1. The "area" of each primitive cell is c^2 (or $[2n-1]^2$), e.g., for c=7, it is 49.

2. The number of divisors p in a primitive cell is p (more generally c divisors for each c), e.g., the number of divisors for c=7 is 7, for c=15, it is 15.

3. In each primitive cell there are "empty" columns (and empty rows). That is there are rows and columns which contain no integers which are $\equiv 0 \pmod p$ for that value of p.

4. The ratio of occupied spaces to total spaces in a primitive cell starts at 3/9=1/3 for c=p=3 and approaches zero asymptotically for p (or c or n) very large.

5. The ratio of empty columns to the total number of columns in a primitive cell is (c-1)/2 and approaches 0.5 for c very large (this applies to rows also).

6. There is a characteristic pattern at the center of each primitive cell consisting of three adjacent positions occupied by divisors which is shaped like an L (rotated 180° in the plane of the Table). See page 13.

7. It does not appear unreasonable to assume that each primitive cell repeats endlessly throughout the infinite array.

8. Structure in the way primes are distributed also occurs in the rows and is about as pronounced as that which appears in the columns.

PROOFS INDICATING UNLIMITED REPETITION OF PRIMITIVE CELLS

The question naturally arises as to whether, from what is observed in Tables 4, 5, and 6, the primitive cells actually do repeat endlessly. The question is not completely answered here but a number of proofs are given that indicate that they do. For example, it is shown immediately below, with the aid of Table 7, that it is true for the primitive cell of 5. We can break up a typical equation for a column into two parts, $x(x-1)$ and c. If we substitute $x=1,2,3,\ldots$.. in the first part, we get $1(0),2(1),3(2),4(3),\ldots$.. or $0,2,6,12,20,30,42,56,\ldots$.. If we retain only the digits parts of these numbers, the result is $0,2,6,2,0,0,2,6,2,0,\ldots$.... a pattern of five digits which repeats endlessly. If we now add $c=1$ to each of these (again retaining only the digits parts) we get $1,3,7,3,1,1,3,7,3,1,\ldots$ another endlessly repeating sequence. Adding $c=3$ to the repeating pattern $0,2,6,2,0$, gives a repeating pattern for the second column, namely $3,5,9,5,3$, and similar repeating patterns occur for $c=5,7,9,\ldots$. as shown in Table 7. But the five repeating sequences $1,3,7,3,1$; $3,5,9,5,3,\ldots$.. of the first five columns (within the first five rows) repeat for <u>every</u> subsequent set of five columns. It is apparent that the patterns for these first five columns repeat without end across the first five rows and these repeat endlessly down <u>all</u> columns. Only five of the spaces in the primitive cell have 5 as divisors. Therefore, the primitive cell for 5 does repeat indefinitely in the c and r directions.

Equations have been developed to prove that the primitive cell for the prime divisor 3 is repeated endlessly throughout the array of Table 1. These are a special case of the most general *equations* which are:*

1. $[(2pn+i) + (pk+m)]^2 - [pk+m] \quad m=-(p-1)/2$

   ....
2. $[(2pn+i)+pk]^2-pk$

   ....
3. $[(2pn+i) + (pk+m)]^2 - [pk+m], \quad m=(p-1)/2$

m, i, n, and k vary, respectively, as follows:

$m =-(p-1)/2,\ldots-2,-1,0,1,2,\ldots+(p-1)/2$; and $i=1,3,5,\ldots 2p-1$; and $n=0$, $k=0,1,2,3,\ldots$; $n=1$, $k=0,1,2,\ldots$(or $k=-1,0,1,2,\ldots$); $n=2$, $k=-1,0,1,\ldots$(or $k=-2,-1,0,1,2,\ldots$ or whatever initial value of k is applicable).

Now for $p=3$, we find that m varies as $-(3-1)/2,0,+(3-1)/2$ or $-1,0,1$ and the equations reduce to the following three:

4. $[(6n+i) + (3k-1)]^2 - [3k-1]$

5. $[(6n+i) + 3k)]^2-3k$

6. $[(6n+i) + (3k+1)]^2-[3k+1]$

---

*See Appendix B for derivation.

For c=3, 2p-1=2(3)-1=5 which makes i=1,3,5 where i=1 applies to the first column in the array and i=3,5 applies to the 2nd and 3rd columns respectively. For n=0 these equations apply to the first three columns no matter how far extended. For n=1,2,3,.... they apply to the second set of three columns, the third set of three, the fourth set of three....respectively, and for all subsequent sets of three for values n>4. Equations 4, 5, and 6, as carried out for five columns and nine rows, and modified to show that they are or are not ≡0(mod 3), are shown in Table 8 (actual substitutions for explicit values of k and n are indicated but not carried out). The primitive cell for the divisor 3 is shown in the heavily outlined part (upper left hand corner) of the table. Those integers in the primitive cell which are common to the intersections of column 1 and row 2, i.e., c1r2 and also c2r2 and c2r3 are equivalent to 0(mod 3). This, of course, holds true for the other five repeated sets of cells or rotated "Ls" of the primitive cell; for example c1r5, c2r5, and c2r6. Thus 18 of the 45 locations shown in Table 8 are ≡0(mod 3). The remaining 27 locations yield integers which are not ≡0(mod 3). This is true for all translations of the primitive cell. (An analysis similar to that for the columns was done for the rows but is not included here as it would be somewhat redundant).

It should be evident that the equation for p=7, derived from the basic set (Equations 1 through 3) would require 7 equations for each column or 49 equations for the primitive cell of 7; for p=41, 1681 equations would be necessary and for c=2n-1 the number required is $(2n-1)^2$ to show that its primitive cell repeats indefinitely. This entails a prohibitive number of equations for n very large.

One additional characteristic of the primitive cell noted previously should be clarified. It is the existence of the three adjacent divisors appearing in the shape of an L (rotated 180°), at the center of each primitive cell the integers of which are ≡0(mod c) for every c. This is proved as follows: In x^2-x+c and x^2+x-r let c=r and let x=c. Then c^2-c+c $(=r^2+r-r)$ = $I_c=I_r\equiv 0$ (mod c,r). This, for all values of c, defines what can be called the main diagonal of the array of Table 1. This diagonal forms a bound on the maximum number of successive prime integers which can exist and these occur for the values x=1 to x=c-1. The next integer below that for which x=c is x=c+1. Substituting this in x^2-x+c gives $I = (c+1)^2-(c+1)+c=c(c+2)\equiv 0$(mod c). The integer to the left of x=c(=r) is r-1 and substituting again for x, $(r-1)^2+(r-1)-r=r(r-2)\equiv 0$ (mod r) and since r=c it is ≡0(mod c). QED

Not only does the "L" of divisors exist in each primitive cell but a parabola exists for which the "L" is the apex and pairs of points symmetrically placed with respect to the curve's axis and at increasingly greater distances from the "L" or apex, make up the balance of the parabola. For the primitive cells of 3 and 5, only the apices are contained within their respective cells; but some of the divisors of diagonally contiguous cells (diagonal translations of the primitive cell) serve as additional points for the curve. Table 6 includes a parabola drawn through a typical cell and adjacent ones. This cell contains five points of the parabola. As c=p increases, more pairs of points are contained in a primitive cell (and in its translations). If parabolas are drawn through every apex (apex of a primitive cell and every translation of it) the individual divisors throughout the array serve as common points for 2,3,4,...,n parabolas. Typical equations which define the first pair of points

TABLE 8. EQUATIONS WHICH PROVE THAT THE PRIMITIVE CELL OF 3 REPEATS THROUGHOUT THE SPECIAL ARRAY OF TABLE 1 NO MATTER HOW FAR EXTENDED

c =	1	3	5	7	9
r =					
1	$3[3(k+2n)^2+k+4n]+1$ n=0, k=0	$3[3(k+2n)^2+3k+8n]+5$ n=0, k=0	$3[3(k+2n)^2+11k+24n]+35$ n=0, k=−1	$3[3(k+2n)^2+k+4n]+1$ n=1, k=−1	$3[3(k+2n)^2+3k+8n]+5$ n=1, k=−1
3	$3[3(k+2n)^2+3k+8n+1]*$ n=0, k=0	$3[3(k+2n+1)^2-k]*$ n=0, k=0	$3[3(k+2n)^2+7k+16n]+17$ n=0, k=0	$3[3(k+2n)^2+3k+8n]+1*$ n=1, k=−1	$3[3(k+2n+1)^2-k]*$ n=1, k=−1
5	$3[3(k+2n)^2-k]+1$ n=0, k=1	$3[3(k+2n)^2+7k+16n+5]*$ n=0, k=0	$3[3(k+2n)^2+9k+20n]+25$ n=0, k=0	$3[3(k+2n)^2-k]+1$ n=1, k=0	$3[3(k+2n)^2+7k+16n+5]*$ n=1, k=−1
7	$3[3(k+2n)^2+k+4n]+1$ n=0, k=1	$3[3(k+2n)^2+3k+8n]+5$ n=0, k=1	$3[3(k+2n)^2+11k+24n]+35$ n=0, k=0	$3[3(k+2n)^2+k+4n]+1$ n=1, k=0	$3[3(k+2n)^2+3k+8n]+5$ n=1, k=0
9	$3[3(k+2n)^2+3k+8n+1]*$ n=0, k=1	$3[3(k+2n+1)^2-k]*$ n=0, k=1	$3[3(k+2n)^2+7k+16n]+17$ n=0, k=1	$3[3(k+2n)^2+3k+8n+1]*$ n=1, k=0	$3[3(k+2n+1)^2-k]*$ n=1, k=0
11	$3[3(k+2n)^2-k]+1$ n=0, k=2	$3[3(k+2n)^2+7k+16n+5]*$ n=0, k=1	$3[3(k+2n)^2+9k+20n]+25$ n=0, k=1	$3[3(k+2n)^2-k]+1$ n=1, k=1	$3[3(k+2n)^2+7k+16n+5]*$ n=1, k=0
13	$3[3(k+2n)^2+k+4n]+1$ n=0, k=2	$3[3(k+2n)^2+3k+8n]+5$ n=0, k=2	$3[3(k+2n)^2+11k+24n]+35$ n=0, k=1	$3[3(k+2n)^2+k+4n]+1$ n=1, k=1	$3[3(k+2n)^2+3k+8n]+5$ n=1, k=1
15	$3[3(k+2n)^2+3k+8n+1]*$ n=0, k=2	$3[3(k+2n+1)^2-k]*$ n=0, k=2	$3[3(k+2n)^2+7k+16n]+17$ n=0, k=2	$3[3(k+2n)^2+3k+8n+1]*$ n=1, k=1	$3[3(k+2n+1)^2-k]*$ n=1, k=1
17	$3[3(k+2n)^2-k]+1$ n=0, k=3	$3[3(k+2n)^2+7k+16n+5]*$ n=0, k=2	$3[3(k+2n)^2+9k+20n]+25$ n=0, k=2	$3[3(k+2n)^2-k]+1$ n=1, k=2	$3[3(k+2n)^2+7k+16n+5]*$ n=1, k=1
19	--------------- n=0, k=3	--------------- n=0, k=3	--------------- n=0, k=2	--------------- n=1, k=2	--------------- n=1, k=2
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-

The equations which determine the primitive cell for the divisor 3 are heavily outlined.

Those equations marked with asterisks are ≡0(mod 3) and this applies to those locations which are translations of the primitive cell.

located next to the apex of a given parabola are $c_1(c_1+4)$ and $c_1(c_1-4)$. Their locations are given by the integers located at $\overline{c_2 r_4}$ and $\overline{c_4 r_2}$, respectively. The next pair of points is given by $c_1(c_1+6)$ and $c_1(c_1-6)$ which are located at $\overline{c_4 r_7}$ and $\overline{c_7 r_4}$, respectively. For these pairs of points, c_1 is the column which contains the apex of L of the parabola; and columns to the left of it are numbered $c_2, c_3, c_4, \ldots$; r_1 is the row in which the apex is located and rows below it are $r_2, r_3, r_4, \ldots$. For example, $\overline{c_4 r_7}$ indicates the point on the curve (i.e., the divisor of the integer) at the intersection of column 4 and row 7 where c_1 is the first column. The general equations for locating points on a parabola are $(c_1+[2n+2])$ and $(c_1-[2n+2])$, $n=1,2,3,\ldots$ and the locations are, respectively, $\overline{c_j r_{j'}}$ and $\overline{c_{j'} r_j}$; $j=(n^2-n+2)/2$ but $n=1,2,3,\ldots$ for j and $n=2,3,4,\ldots$ for j'.

The equation for the parabola shown superposed in Table 6 has the form $x^2+x+y^2+y-2xy$. Its origin is at the exact center of the primitive cell. The sets of coordinates used were $x,y=(0,0)$, $(-1,0)$, $(0,-1)$, $(-3,-1)$, $(-1,-3)$, $(-3,-6)$, and $(-6,-3)$. Other parabolas constructed (but not shown) were for $p=3,5,11,\ldots,23$. The same coordinates were used for each of them. It is reasonable to assume that identical parabolas can be constructed for all primitive cells and for all translations of same.

There are five known columns for which the maximum number of unbroken primes occur. The one best known is that shown in column 21, x^2-x+41 (Tables 1 and 2). The other four, the first two of which can be considered trivial, are x^2-x+3, x^2-x+5, x^2-x+11, and x^2-x+17.

ANOTHER PRIME NUMBER SIEVE

The additional possible method, alluded to earlier, for searching for prime-rich equations is based on setting up tables similar to those of Tables 1, 4, 5, and 6. This can be described as follows: Imagine a very large array like that of Table 1. Next imagine a large array, based on Table 4, which is transparent and has the primitive cell for 3 repeated indefinitely in both the column and row directions. This "Table 2" can overlay "Table 1". Additional similar arrays for the primitive cells of $5,7,11,13,\ldots p$ (p=prime) can be imagined to successively overlay imaginary Tables 1 and 2. This set of an original array plus all repeating primitive cell overlays forms a new type of sieve for finding prime numbers. What can and should be given further study is where coincidences of "empty" columns occur (columns for each divisor p in which no divisors occur) with all the various overlays in place on the imaginary Table 1-type array. It can be seen from Table 1 that there is one outstanding coincidence of "empty" columns--that for c=41. Here the empty columns for the first 11 primes $(3,5,7,\ldots,37)$ occur at the 21st column. The first composite number which breaks the string of unbroken primes is, of course, $c^2=41^2=1681$. A computer program to implement, in a practical way, the imagined Table 1-type array with overlays, within practical limits, should not be too difficult to set up to look for any prime-rich equations which may exist other than those found by Euler et al.

There are undoubtedly a number of other interesting characteristics of the new array not described in this article that will be found. It is hoped that this article will spur further searches in this area.

CONCLUSIONS

A simple type of diagonal array of all the positive odd integers can be made such that all its columns are described by the equations x^2-x+c (and its rows by x^2+x-r) where $c=1,3,5,\ldots,2n-1,\ldots$ ($r=1,3,5,\ldots$).

A related array can be derived from the above array for each prime value, p, of c such that the only elements in this array consist of the divisors p where p has been substituted for each I which is $\equiv 0 \pmod p$ and all other integers in the first array not $\equiv 0 \pmod p$ are left blank. All the divisors of the derived array occur in a fixed structure which consists of what is called a "primitive" cell and all its translations. This fixed structure can also be described as that of a set of parabolas each of the form $x^2+x+y^2+y-2xy$ (origin $x=y=0$ at the apex of the parabola) which make angles of 45° with the "c" and "r" directions of the array. All points in this array are contained in the parabolas and, in general, most and perhaps all points are parts of 2,3,4,.... parabolas.

Prime-rich equations of the type Euler discovered (e.g., x^2-x+41) tend to occur for the criteria c=prime, $x_1 \equiv 0 \pmod 3$ where x_1 is the first value of x for each column (x^2-x+c). This is because a column for which c is composite tends to have a larger number of divisors than one which has c=prime. Moreover, where $x_1 \equiv 0 \pmod 3$ these columns are ones which contain no integers $\equiv 0 \pmod 3$. Because of the nature of the array's structure, all other columns where x_1 is not $\equiv 0 \pmod 3$ have 3's as divisors.

A set of arrays consisting of the primitive cells and their translations superposed on the diagonal array of all the odd integers forms another sieve for finding prime numbers.

The picture of the new array and its interrelationships with its accompanying primitive cell arrays show how and why there is a non-random structure in the way prime numbers occur in the number system.

REFERENCES

1. Gardner, Martin, "Mathematical Recreations," Scientific American, Vol. 210, No. 3, Mar 1964, pp. 120-127.

APPENDIX A

ADDITIONAL OBSERVATIONS ON THE STRUCTURE OF THE SPECIAL DIAGONAL ARRAY AND PRIMITIVE CELL ARRAYS

A few other observations not pointed out above are added here.

1. The "L" shaped triad of divisors which is at the center of each primitive cell overlaps the "L" of the next adjacent primitive cell. For example in Table 1, for the primitive cell of 3, the integers are 3,9,15 (i.e., at locations $\overline{c1r2}$, $\overline{c2r2}$, and $\overline{c2r3}$).* These overlap the integers for the primitive cell of 5, namely 15, 25, 35 (i.e., locations $\overline{c2r3}$, $\overline{c3r3}$, and $\overline{c3r4}$).* Note also that 3 and 5 are divisors of 15. This is why there are always two composite numbers following columns with unbroken strings of primes such as for c=3, c=5, c=11, c=17, and c=41.

2. In conjunction with the above, it should be noted that c=41 and c=43 are twin primes. Since the primitive cell for 3 and all translations of it have two columns with periodic occurrences of the divisor 3 and the third column with no 3's, it is evident that the column for c=41 is empty of the divisor 3 (and, as was pointed out earlier, of the divisors 5,7,11,....,37). Compare Tables 4 and 1. It is also easily seen that the column next to c=41, i.e., c=43 must be a column which has recurring 3's as divisors. This is true of a number of other such pairs having unbroken strings of primes for x_1=1,2,3,....., c-1, e.g., 3,5,; 5,7; 11,13 and 17,19. The first two of these pairs are trivial for the unbroken strings of primes are very short. The first string has only the two primes 3 and 5, the second has four primes 5, 7, 11, and 17. Thus the probabilities are quite high that a prime-rich column will be followed by a prime-poor one.

3. Though it is not immediately obvious why values of c which are prime are more likely to be prime-rich, it is easier to see why those columns having composite values of c are not, for example c=15. As can be seen by looking at Table 1, this column has recurring integer couples ≡0(mod 5) such as 105,125; 225,255;.... In addition, it contains recurring integers which are ≡0(mod 3), e.g., 87,105. Of course, some integers are divisible by both 3 and 5. In any event, many such columns are prime-poor.

c=15 represents one member of a subset of the set of all composite values of c. This subset is that for which c=15,25,35,..... i.e. 10n+5, n=1,2,3,.... . The column for c=15 has a density of primes (for 40 rows) of 10%. The density of primes in any member of this subset can be at most 60% but is usually is considerably less because of other divisors. A prominent member of this set is that of c=95 (x^2-x+95) which has a density of 50% (see Table 3).

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*Note: $\overline{c1r2}$ indicates the integer at the intersection of the first column and the second row; $\overline{c2r3}$ the integer at column 2 row 3 etc.

A more striking subset is that for which c=9,15,21,....i.e. 6n+3, n=1,2,3,. .... where the maximum density of primes cannot exceed 33%. All subsets of composite values of c are described by p(2n+1), n=1,2,3,....; p is prime.

4. Now it can be seen why a correlation exists between prime-rich equations (columns) and the two criteria c is prime and x_1=0(mod 3). The columns of Table 1 can be divided into three sets, those for which the first value, x_1, to be substituted in the equation is 3n, n=1,2,3,.... and those for which x_1=3n+1 and 3n+2, n=0,1,2,3,.... . Of the three sets, only x_1=3n is equivalent to 0(mod 3) and that for all n. If c is prime for any one of the columns for which x_1=3n, that column contains no integers divisible by 3 and thus is more likely to be prime-rich than if c is prime for other columns of the remaining two sets; for which it is obvious from Tables 1 and 4 that every column of these sets contains integers ≡0(mod 3). Of course, one other consideration determines how prime-rich a column which fulfills the criteria can be, and that is what other divisors apply to the integers of that column, e.g., the third column, x^2-x+5 where 5 is prime and x_1=3≡0(mod 3).

5. If the sets of equations x^2-x+c=y and x^2+x -r=y are plotted graphically they form two sets of nested parabolas. The ones involving c have a common axis x=1/2 with apices at (x=1/2; y=3/4, 2 3/4 3 3/4.....). The ones involving r have a common axis of symmetry x=-1/2 and apices at (x=-1/2; y=-1 1/4 -3 3/4, -5 3/4,.....). The left branches of both sets do not intersect; however the right branches do and the points at which they intersect make up the integral values of Table 1. Certain sets of these points form subsets of the totality of points (intersections). These sets are made up of those integers which are ≡0(mod c)- [or (mod r)]. Each of these subsets c=1,3,5,.....,2n-1 if plotted as an array such as those of Tables 4, 5 and 6 can be described as sets of parabolas (physically traceable in the tables) with apices in the centers of the primitive cell and all its translations (as described on page 13). Each parabola can be represented by an equation of the form x^2+x+y^2+y-2xy where the origin has been chosen to be x=y=0; the axis of each parabola is on a line, x=y, which is perpendicular to the main diagonal (see pages 9 and 15) of the array of Table 1.

One can consider the plotted points at all intersections as being transformed from their positions as points on intersecting parabolas into the linear forms (i.e., columns) of Table 1. The subsets of those points (those which are =0(mod c)) are transformed from what - in a preliminary analysis - appear to be linear arrays of points on the plotted graphs of intersecting parabolas into parabolic curve arrays of points on the primitive cell array type of table such as those of Tables 4, 5 and 6.

APPENDIX B

DERIVATION OF EQUATIONS SHOWING WHY PRIMITIVE CELLS TRANSLATE INDEFINITELY

The derivation of the equations to show that primitive cells translate indefinitely in the c and r directions of the array in Table 1 is as follows: Substituting c for x in $I_c=x^2-x+c$ gives $c^2-c+c\equiv 0 \pmod{c}$. The n^{th} integer below c is obtained by substituting c+n for x, i.e., $I_{c+n}=(c+n)^2-(c+n)+c=(c+n)^2-n$. Next, let limiting expressions pk-m and pk+m replace n where, (1) p is the divisor sought, (2) $m=(p-1)/2$, and (3) c can be equal to p but will take on other non-prime values. In order to show this let $c=2pn+i$, where i for each primitive cell takes on values 1,3,5,..., p,....,2p-1 and also where m takes on the values $-(p-1)/2,\ldots,-2,-1,0,1,2,\ldots,+(p-1)/2$. Then the most general equations take the form, i.e., $[(2pn+i) + (pk-m)]^2 - (pk-m)$, etc.

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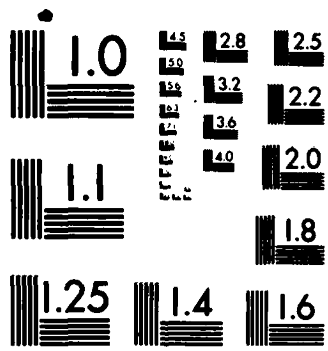
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Change 1
6 Sep 1985
1 page (s)

This publication is changed as follows:

Add the following information to page 15, after the first paragraph:

A better description of the "parabolic" arrangement of the integers divisible by p, referred to on this page, is given by the following equation:

7. $I=(n+m)2px- [(\{n+m\}p)^2 + (n-\{m+1\}p]$. Two initial values of x are given by:

8. $x=p(n+m)$, $p(n+m)+1$ which are for, respectively, the apex of the parabola (of the rotated L or Triad of divisors mentioned earlier) and the next integer horizontally adjacent to it. For n=1 m=0 (any value of p) 7. and 8. describe the parabolas for the primitive cells. But for n=1,2,3,.... for each value of m, m=1,2,3,.... they describe all translations of each primitive cell. The equation 7. is a generalization of the Diophantine equations such as $I=6x-9$, (x=2,3,4, & 5); $I=12x-33$, (x=5, 6, 7, etc.) which describe the straight lines formed by the intersections of the intersecting sets of parabolas mentioned in 5. of page A-2, Appendix A.

Note that 7. is linear in x as m= 2p(n+m) and b is equal to the negative term. Also p can be factored out of 7; therefore, $I\equiv 0 \pmod p$ for all combinations of values of m and n.

Equations 7 and 8 are a step closer to a proof that the configuration of integers in every cell repeats without limit in the "c" and "r" directions.

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